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UPPER AND LOWER BOUNDS FOR THE SKEW HERMITIAN RANDIC ENERGY OF STANDARD GRAPHS

N. Keerthana [1] and S. Meenakshi [2]*

[1] Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Pallavaram, Chennai, India. Email: nkeerthanamphil@gmail.com
[2] Associate Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Pallavaram, Chennai, India.
*Corresponding Author Email: meenakshikarthikeyan@yahoo.co.in

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Abstract

Let us consider a simple graph G. The energy of the graph is defined as the sum of the absolute values of the eigen values of the adjacency matrix [1] & [2]. The energy of $R_{SH}^{*}(G)$, denoted by $E_{R_{SH}^{*}}(G)$, is called Skew- Hermitian Randić energy, which is defined as the sum of the absolute values of its eigenvalues of $R_{SH}^{*}(G)$, that is, $E_{R_{SH}^{*}}(G) = \sum_{i=1}^{p} |\rho_i|$[9]. The total π electron energy of conjugated hydrocarbon molecules are closely connected with graph invariant[10],[11]. Recently based on the eigen values of graph matrices various energies are computed. For a graph matrix, we can determine the eigen values based on which we can compute the energy of the graph. In this paper, we have determined the Skew-Hermitian Randic Energy of some standard graphs[13],[14],[15].

Keywords: Skew-Hermitian Randic matrix, Skew-Hermitian Randic Energy, Mixed graph, Wheel graph, Cycle graph, Path graph

1. INTRODUCTION

It was Ivan Gutman, mathematician, in 1978 who first defined and introduced the Energy of a graph. But the motivation for his definition appeared earlier in 1930's Erich Huckel proposed the Huckel Molecular Orbital Theory [1], [7] & [8]. The energy of a graph G, denoted by $E(G)$ is defined to be the sum of the absolute value of the eigen values of the adjacency matrix. This graph invariant is closely connected to a chemical quantity known as the total π electron energy of conjugated hydrocarbon molecules [3] & [4]. The study of Graph Energy developed first in the field of chemistry. Chemists used Huckel's method to approximate energies associated with π – electron orbital's in a special class of molecules called conjugated hydrocarbons. Gutman first introduced the concept of 'energy of graph' for a simple graph. At first very few mathematicians seemed to be interested in this concept [5]. However, over the years graph energy has become an interesting area of research for mathematicians and several variations have been introduced [6] & [16].

Research in graph energy and its numerous variants shows lot of attention nowadays [12]. In view of this, we found it purposeful to present data on the enormous increase of work in this area. In addition, we outline the various, sometimes quite unexpected and surprising, applications that graph energies have found in other fields of science.

2. PRELIMINARIES

The following are the necessary prerequisites required for the study of this paper

Definition 1

The energy of $R_{SH}^{*}(G)$, denoted by $E_{R_{SH}^{*}}(G)$, is called ***Skew- Hermitian Randić energy,*** which is defined as the sum of the absolute values of its eigenvalues of $R_{SH}^{*}(G)$, that is, $E_{R_{SH}^{*}}(G) = \sum_{i=1}^{p} |\rho_i|$.

Definition 2

Let G be a mixed graph on the vertex set $\{v_1, v_2, \dots, v_n\}$, then the ***Skew- Hermitian Randic matrix of G*** is the $p \times p$ matrix $R_{SH}^{*}(G) = -((r_{sh})_{mn})$, where

$$R_{SH}^{*}(G) = -((r_{sh})_{mn}) = -\begin{cases} \dfrac{1}{\sqrt{d_m d_n}}, & \text{if } v_m \leftrightarrow v_n, \\ \dfrac{\mathrm{i}}{\sqrt{d_m d_n}}, & \text{if } v_m \rightarrow v_n, \\ \dfrac{-\mathrm{i}}{\sqrt{d_m d_n}}, & \text{if } v_n \rightarrow v_m, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 3

Let G be a mixed graph of order p with its ***Skew- Hermitian Randić matrix*** $R_{SH}^{*}(G)$. Denote the R_{SH}^{*}***-characteristic polynomial*** of $R_{SH}^{*}(G)$ of G that is for every $x_{ij} \in G$.

$$P_{R_{SH}^{*}}(G, a) = -(a^p + x_1 a^{p-1} + x_2 a^{p-2} + \cdots + x_n)$$

$$P_{R_{SH}^{*}}(G, a) = -a^p - x_1 a^{p-1} - x_2 a^{p-2} - \cdots - x_n$$

where $(1 \leq i, j \leq p)$.

Definition 4

In graph theory, a ***mixed graph*** G = (V, E, A) is a graph consisting of a set of vertices V, a set of (undirected) edges E, and a set of directed edges (or arcs) A.

Definition 5

A path graph denoted as P_n is a graph with n vertices in which the initial and the final vertex is of degree 1 and the remaining $n - 2$ vertices are of degree 2.

Definition 6

A cycle graph, denoted as C_n is a graph with n number of vertices and where the initial and the final vertex is the same. A graph without cycles is called an acyclic graph.

Definition 7

A ***wheel graph*** is a graph formed by connecting a single universal vertex to all vertices of a cycle.

3. MAIN RESULTS

Theorem 1

Let $P_n = G$ be the mixed path graph. The upper and lower bounds of Skew-Hermitian Randic Energy of the mixed Path graph is given by $-(\frac{ni}{2}+\frac{1}{\sqrt{2}}) \leq R_{SH}^{*}(G) \leq (\frac{ni}{2}+\frac{1}{\sqrt{2}})$

Proof.

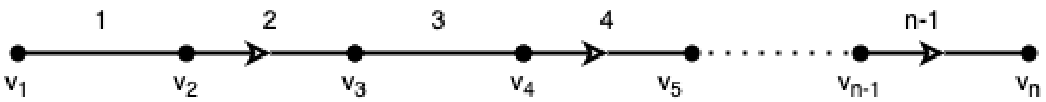

Figure 1: Mixed Path Graph P_n

Let $P_n = G$ be the mixed path graph. Let V be the set of vertices.

$V = \{v_1, v_2, v_3, \dots v_n\}$

E be the set of undirected edges, that is, $E = \{1,3,5, \dots n-2\}$.

A be the set of directed edges, that is, $A = \{2,4,6, \dots n-1\}$.

The Skew Hermitian Randic Matrix of G is given by

$$R_{SH}^{*}(G) = -\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \dots & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{2} & \dots & 0 \\ 0 & \frac{-i}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{-i}{\sqrt{2}} \end{pmatrix}$$

$$R_{SH}^{*}(G) = \begin{pmatrix} 0 & \frac{-1}{\sqrt{2}} & 0 & \dots & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-i}{2} & \dots & 0 \\ 0 & \frac{i}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{i}{\sqrt{2}} \end{pmatrix}$$

The characteristic polynomial for the above matrix is $\det(\lambda I - R_{SH}^{*}(G))$ where I denotes the Identity matrix of G.

$$P_{R_{SH}^*}(G,a) = \begin{vmatrix} \lambda & 0 & 0 & \dots & 0 \\ 0 & \lambda & 0 & \dots & 0 \\ 0 & 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & \dots & \lambda \end{vmatrix} - \begin{vmatrix} 0 & \frac{-1}{\sqrt{2}} & 0 & \dots & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-i}{2} & \dots & 0 \\ 0 & \frac{i}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{vmatrix}$$

$$P_{R_{SH}^*}(G,a) = \begin{vmatrix} \lambda & \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ \frac{1}{\sqrt{2}} & \lambda & \frac{i}{2} & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & \lambda & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \lambda \end{vmatrix} \quad R_i \to R_i - (R_3 + \tfrac{1}{\sqrt{2}}),\ i = 3,4,5 \dots n$$

The transformed matrix is

$$P_{R_{SH}^*}(G,a) = \begin{vmatrix} \lambda & \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ 0 & \lambda + \frac{i}{2} + \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & \lambda & \frac{1}{2} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \lambda \end{vmatrix} \quad C_2 \to C_2 + iC_4$$

The transformed matrix is

$$P_{R_{SH}^*}(G,a) = \begin{vmatrix} \lambda & \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ 0 & \lambda + \frac{i}{2} + \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ 0 & 0 & \lambda & \frac{1}{2} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \lambda \end{vmatrix}$$

The characteristic polynomial is given by $P_{R_{SH}^*}(G,a) = \lambda\left(\lambda + \frac{i}{2} + \frac{1}{\sqrt{2}}\right)\lambda^{n-2}$

$$P_{R_{SH}^*}(G,a) == \lambda^n + \left(\frac{ni}{2} + \frac{1}{\sqrt{2}}\right)\lambda^{n-2}$$

$$P_{R_{SH}^*}(G,a) = \lambda^{n-1}\left(\lambda + \frac{ni}{2} + \frac{1}{\sqrt{2}}\right) = 0$$

$P_{R_{SH}^*}(G,a) = \lambda^{n-1} = 0 \Rightarrow \lambda = 0\ (n-1)$ times.

$$\lambda = -(\tfrac{ni}{2} + \tfrac{1}{\sqrt{2}})$$

Hence, the upper and lower bounds of Skew Hermitian Randic Energy of the mixed Path graph is given by $-(\frac{ni}{2} + \frac{1}{\sqrt{2}}) \le R_{SH}^*(G) \le (\frac{ni}{2} + \frac{1}{\sqrt{2}})$

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Theorem 2

Let $C_n = G$ be the mixed cycle graph. The upper and lower bounds of Skew-Hermitian Randic Energy of the mixed Cycle graph is given by $-\frac{1}{2} \leq R_{SH}^{*}(G) \leq \frac{1}{2}$

Proof.

Let $C_n = G$ be the mixed cycle graph. Let V be the set of vertices.

$V = \{v_1, v_2, v_3, \dots v_n\}$

E be the set of undirected edges, that is, $E = \{1,3,5, \dots n-2\}$.

A be the set of directed edges, that is, $A = \{2,4,6, \dots n-1\}$.

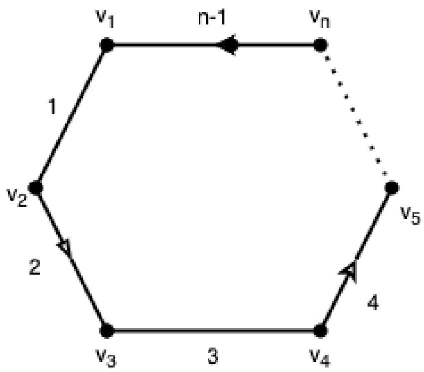

Figure 2: Mixed Cycle graph C_n

The Skew Hermitian Randic Matrix of G is given by

$$R_{SH}^{*}(G) = -\begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \dots & 0 \\ \frac{1}{2} & 0 & \frac{i}{2} & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & 0 & \frac{1}{2} & \dots & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$R_{SH}^{*}(G) = \begin{pmatrix} 0 & \frac{-1}{2} & 0 & 0 & \dots & 0 \\ \frac{-1}{2} & 0 & \frac{-i}{2} & 0 & \dots & 0 \\ 0 & \frac{i}{2} & 0 & \frac{-1}{2} & \dots & 0 \\ 0 & 0 & \frac{-1}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

The characteristic polynomial for the above matrix is $\det(\lambda I - R_{SH}^{*}(G))$ where I denotes the Identity matrix of G.

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & 0 & 0 & \dots & 0 \\ 0 & \lambda & 0 & \dots & 0 \\ 0 & 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & \dots & \lambda \end{vmatrix} - \begin{vmatrix} 0 & \frac{-1}{2} & 0 & 0 & \dots & 0 \\ \frac{-1}{2} & 0 & \frac{-i}{2} & 0 & \dots & 0 \\ 0 & \frac{i}{2} & 0 & \frac{-1}{2} & \dots & 0 \\ 0 & 0 & \frac{-1}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{vmatrix}$$

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & \frac{1}{2} & 0 & 0 & \dots & 0 \\ \frac{1}{2} & \lambda & \frac{i}{2} & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & \lambda & \frac{1}{2} & \dots & 0 \\ 0 & 0 & \frac{1}{2} & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix}$$

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & \frac{1}{2} & 0 & 0 & \dots & 0 \\ \frac{1}{2} & \lambda & \frac{i}{2} & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & \lambda & \frac{1}{2} & \dots & 0 \\ \frac{-1}{2} & \frac{-1}{2} & 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} R_2 \to R_2 + R_4$$

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & \frac{1}{2} & 0 & 0 & \dots & 0 \\ 0 & \lambda - \frac{1}{2} & \frac{i}{2} & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & \lambda & \frac{1}{2} & \dots & 0 \\ \frac{-1}{2} & \frac{-1}{2} & 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} C_2 \to C_2 - C_1$$

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & \frac{1}{2} - \lambda & 0 & 0 & \dots & 0 \\ 0 & \lambda - \frac{1}{2} & \frac{i}{2} & 0 & \dots & 0 \\ 0 & \frac{-i}{2} & \lambda & \frac{1}{2} & \dots & 0 \\ \frac{-1}{2} & 0 & 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} C_1 \to C_1 + C_4$$

$$P_{R_{SH}^{*}}(G,a)=\begin{vmatrix} \lambda & \frac{1}{2}-\lambda & 0 & 0 & \dots & 0\\ 0 & \lambda-\frac{1}{2} & \frac{i}{2} & 0 & \dots & 0\\ 0 & \frac{-i}{2} & \lambda & \frac{1}{2} & \dots & 0\\ \frac{-1}{2} & 0 & 0 & \lambda & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} R_3 \to R_3 + R_4$$

$$P_{R_{SH}^{*}}(G,a)=\begin{vmatrix} \lambda & \frac{1}{2}-\lambda & 0 & 0 & \dots & 0\\ 0 & \lambda-\frac{1}{2} & \frac{i}{2} & 0 & \dots & 0\\ 0 & 0 & \lambda & \frac{1}{2} & \dots & 0\\ \frac{-1}{2} & 0 & 0 & \lambda & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} C_2 \to C_2 + \frac{C_4}{i}$$

$$P_{R_{SH}^{*}}(G,a)=\begin{vmatrix} \lambda & \frac{1}{2}-\lambda & 0 & 0 & \dots & 0\\ 0 & \lambda-\frac{1}{2} & \frac{i}{2} & 0 & \dots & 0\\ 0 & 0 & \lambda & \frac{1}{2} & \dots & 0\\ \frac{-1}{2} & 0 & 0 & \lambda & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} C_5 \to C_5; C_1 \to C_1 - C_5$$

$$P_{R_{SH}^{*}}(G,a)=\begin{vmatrix} \lambda & \frac{1}{2}-\lambda & 0 & 0 & \dots & 0\\ 0 & \lambda-\frac{1}{2} & \frac{i}{2} & 0 & \dots & 0\\ 0 & 0 & \lambda & \frac{1}{2} & \dots & 0\\ 0 & 0 & 0 & \lambda & \dots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{vmatrix}$$

$P_{R_{SH}^{*}}(G,a)=\lambda(\lambda-\frac{1}{2})\lambda\lambda\dots\lambda$

$P_{R_{SH}^{*}}(G,a)=(\lambda-\frac{1}{2})\lambda^{n-1}$

$\Rightarrow \quad (\lambda-\frac{1}{2})\lambda^{n-1}=0$

$$\Rightarrow \lambda^{n-1}=0\,;\ \lambda-\frac{1}{2}=0$$

$$\lambda=0(n-1)times\,;\lambda=\frac{1}{2}$$

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The upper and lower bounds of Skew Hermitian Randic Energy of the mixed Cycle graph is given by $-\frac{1}{2} \leq R^*_{SH}(G) \leq \frac{1}{2}$

Theorem 3

Let $W_n = G$ be the mixed wheel graph. The upper and lower bounds of Skew-Hermitian Randic Energy of the mixed star graph is given by $-\frac{1}{\sqrt{n-1}} \leq R^*_{SH}(G) \leq \frac{1}{\sqrt{n-1}}$

Proof.

Let $W_n = G$ be the mixed star graph. Let V be the set of vertices.

$V = \{v_1, v_2, v_3, \dots v_n\}$

E be the set of undirected edges, that is, $E = \{2, 4, 6, \dots\}$.

A be the set of directed edges, that is, $A = \{1, 3, 5, \dots\}$.

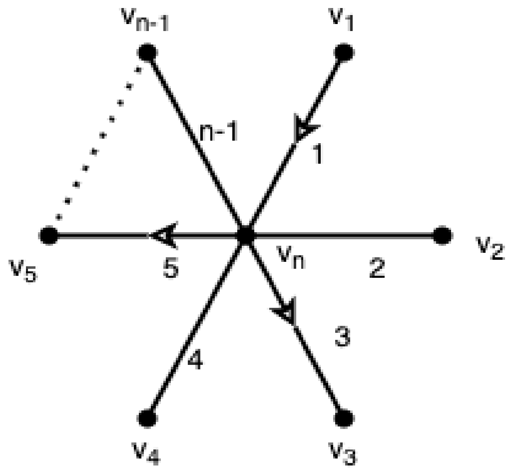

Figure 3: Mixed Star graph S_n

The Skew Hermitian Randic Matrix of G is given by

$$R^*_{SH}(G) = -\begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 & \frac{i}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{-i}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{\sqrt{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{\sqrt{n-1}} & 0 \end{pmatrix}$$

$$R^*_{SH}(G) = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 & \frac{-i}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{i}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{-1}{\sqrt{n-1}} & 0 \end{pmatrix}$$

The characteristic polynomial for the above matrix is $\det(\lambda I - R^*_{SH}(G))$ where I denotes the Identity matrix of G.

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & 0 & 0 & \dots & 0 \\ 0 & \lambda & 0 & \dots & 0 \\ 0 & 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & \dots & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 & 0 & \dots & 0 & \frac{-i}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{i}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{-1}{\sqrt{n-1}} & 0 \end{vmatrix}$$

$$P_{R^*_{SH}}(G,a) = \begin{vmatrix} \lambda & 0 & 0 & 0 & \dots & 0 & \frac{-i}{\sqrt{n-1}} \\ 0 & \lambda & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ 0 & 0 & \lambda & 0 & \dots & 0 & \frac{i}{\sqrt{n-1}} \\ 0 & 0 & 0 & \lambda & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{\sqrt{n-1}} & \lambda \end{vmatrix} C_{n-1} \to C_{n-1} - C_n$$

$$P_{R_{SH}^*}(G,a) = \begin{vmatrix} \lambda & 0 & 0 & 0 & \dots & 0 & \frac{-i}{\sqrt{n-1}} \\ 0 & \lambda & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ 0 & 0 & \lambda & 0 & \dots & 0 & \frac{i}{\sqrt{n-1}} \\ 0 & 0 & 0 & \lambda & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \lambda - \frac{1}{\sqrt{n-1}} & \frac{1}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{\sqrt{n-1}} - \lambda & \lambda \end{vmatrix} R_n \to R_n + R_{n-1}$$

$$P_{R_{SH}^*}(G,a) = \begin{vmatrix} \lambda & 0 & 0 & 0 & \dots & 0 & \frac{-i}{\sqrt{n-1}} \\ 0 & \lambda & 0 & 0 & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ 0 & 0 & \lambda & 0 & \dots & 0 & \frac{i}{\sqrt{n-1}} \\ 0 & 0 & 0 & \lambda & \dots & 0 & \frac{-1}{\sqrt{n-1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \lambda - \frac{1}{\sqrt{n-1}} & \frac{1}{\sqrt{n-1}} \\ 0 & 0 & 0 & 0 & \cdots & 0 & \lambda + \frac{1}{\sqrt{n-1}} \end{vmatrix}$$

$P_{R_{SH}^*}(G,a) = \lambda\lambda\lambda \dots \lambda \left(\lambda - \frac{1}{\sqrt{n-1}}\right)\left(\lambda + \frac{1}{\sqrt{n-1}}\right) = 0$

$P_{R_{SH}^*}(G,a) = \lambda^{n-2} \left(\lambda - \frac{1}{\sqrt{n-1}}\right)\left(\lambda + \frac{1}{\sqrt{n-1}}\right) = 0$

$P_{R_{SH}^*}(G,a) = \lambda^{n-2} = 0;\ \left(\lambda - \frac{1}{\sqrt{n-1}}\right)\left(\lambda + \frac{1}{\sqrt{n-1}}\right) = 0$

$P_{R_{SH}^*}(G,a) = \lambda = 0 (n-2) times;\ \lambda^2 - (\frac{1}{\sqrt{n-1}})^2 = 0$

$$\Rightarrow \lambda = \pm \frac{1}{\sqrt{n-1}}$$

Hence, the upper and lower bounds of Skew Hermitian Randic Energy of the mixed star graph is given by $-\frac{1}{\sqrt{n-1}} \le R_{SH}^*(G) \le \frac{1}{\sqrt{n-1}}$.

4. CONCLUSİON

In this paper, the upper and lower bounds of the Skew Hermitian Randic Energy of standard mixed graphs like Star graph, Cycle graph, Path graph have been computed. This can be extended to various graphs also. The directed arcs of the considered mixed graph can be generalized and hence the generalized Skew Hermitian Randic

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Energy for the considered mixed graph can be determined. As a consequence, various properties can be computed.

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