EFFECTIVENESS OF AN M/M/1 QUEUE WITH ENCOURAGED SYSTEM-SIZE ANALYSIS, WORKING-BREAKDOWN POLICY AND SYSTEM MAINTENANCE

R. Jeyachandhiran [1] and P. Rajendran [2*]

[1,2] Department of Mathematics, School of Advanced Sciences,
Vellore Institute of Technology, Vellore, Indıa.
Email: [1]jeyachandhiran.r@vit.ac.in, [2]prajendran@vit.ac.in. (*Corresponding Author)

DOI: 10.5281/zenodo.12265700

Abstract

We study the effectiveness of an M/M/1 queue with working breakdown policy, and consumer maintenance along with the proposed encouraged system-size analysis in this article. Breakdown occurs only when the primary service provider being at work, It not only stops the primary server, but it also eliminates all existing consumers from the entire system. Once the primary service provider breaks down, It has been moved for repairs. The alternate the service provider offers consumers a slow service working-breakdown service policy ("W-B-S-P") until the primary service provider is repaired. The consumers become impatient (renege) because of the working-breakdown. We develop the balance equations by utilizing the stochastic continuous time Markov-chain. Also, The expected system-size and excepted sojourn-time of a consumer is obtained. Finally, The effect of an encouraged arrival and maintenance through numerical and graphical illustrations are presented.

Keywords: Alternate Server, Encouraged System-Size, Markov-Chain, Impatient Consumers.

1. INTRODUCTION

An M/M/1 queue with working breakdown policy, and maintenance of impatient consumers along with the proposed encouraged system-size analysis in this article. However, in most cases, a service provider's failure does not entirely disrupt a consumer's service. For instance, Existence of malware (viruses) in the system might sluggish down the computer's routine. One more instance is the machine-replacement problems. When the primary operator (main machine) fails, it is instantly swapped by alternative server (substitute machine). The alternative-server serves slower until the primary server is fixed. The idea of working-breakdowns was first presented by [9]. The M/G/1 queue with working-breakdown system-size and sojourn-time distributions were discussed in [10]. queues with impatience consumers have drawn substantial consideration in earlier, where the cause of consumer reneging was too lengthy line previously experienced in queue. [1] analyzed the queue where clients become impatient because of the nonappearance of service provider, more exactly, because of the service provider vacation. [18] studied the M/M/1 queue with server breakdowns where consumers are reneging since the server is inaccessible. [16] developed the M/M/1 non time dependent model with catastrophes and reneging. [13] analyzed the queue where the impatience of consumers happens due to a slower service by the processor. They analyzed Markovian queueing models where single, multiple, infinite service provider in two-stage (slow and fast) Markovian arbitrary environment with impatient consumers. [19] have studied impatience of jobs in working server vacation queueing system where reneging is because of the server working server vacation. [14] discussed the customer impatient Markovian queue with various and solitary working server vacations. Further [11] studied the queueing model with reneging clients which combined the structures of balking clients, Bernoulli server provider vacation and server interruption. The queue featuring disasters is distinguished by the fact that the existence of catastrophes dismisses all current tasks but also fails the server. [17] analyzed the M/M/1 model with catastrophes define the performance of the distributed file systems with site catastrophe. [19] developed an M/M/1 queueing system with catastrophes was stretched to the M/G/1 system. [2] studied the queueing model with negative clients and catastrophes. [12] deliberated the discrete-time queueing system with processor breakdowns, arbitrary repair-epochs. [4] discussed an M/G/1 queueing model in the multiple stage environment with server disasters. Encouraged arrival perception in the queueing model is widely utilized to tempt more customers by providing them with certain discounts to obtain an excellent reaction from consumers; thus, this will undoubtedly raise the volume of consumers in the entire system and improve professional profits. [15, 7] established the M/M/1/N queue with encouraged client arrival. [8] analyzed the encouraged consumer arrival queue under working server vacations. In [3], a time-independent analysis of an M[x]/G/1 queue with dual stages of customer facility and vacation was developed. [5] developed the "F" type service policy in an M/M/1 retrial orbit line with vacations and reneging clients. [6] analyzed the efficiency of encouraged consumer arrival of a general times double phase queueing model with T-policy and Bernoulli vacations. The working-breakdown M/M/1 queue were discussed in [20]. Evaluations regarding the exponential distribution employ the features underlying linear structures with arbitrary coefficients investigated in [21]. The structure of the paperwork is as follows: Section 1 covers introduction. Model description in Section 2. Balance equations in Section 3. Encouraged system-size probabilities in Section 4. Sojourn-time distribution in Section 5. Numerical examples in 6. Result and discussions in Section 7. Conclusion in Section 8.

2. DESCRIPTION OF THE QUEUEING MODEL

We develop the model explanation of a queueing system in this part under the following structures. Consumers reach at a single server system, in accordance with the encouraged arrival process with the rate of $\lambda(1+\varphi)$. Where "φ" represents the discount rate. The primary server's service times are exponentially distributed with the value μ_a, while consumers are serviced based on FCFS queue discipline. Breakdowns occur only while the primary server is on duty. They not just eliminate all current consumers from the service facility, but also cause the primary processor to fail. Let "d" denotes the inter arrival times between the consecutive breakdowns and it follows exponential distribution meant by ζ. When the primary server crashes, the service provider is sent right away in order to start the repair process. The primary server repair times are characterized by an exponential distribution meant by rate γ. The service provider is as better as new following the repair process. While the primary server is being fixed, the alternate server serves consumers. The alternate server serves up the service during the working breakdown, and its service periods are also considered to be exponential distribution denoted as μ_b ($\leq \mu_a$). Throughout a fixed time, the new consumers coming continuously. Upon returning to work after repairs, the primary server detects that there are consumers in the system. The alternative service processor pauses service and the primary server resumes and activates service with rate μ_a. If there are no consumers in the service facility at the termination of the server repair, the primary server back to the system, remain free, and holds for incoming consumers. The consumers expected to be impatient at the time of W-B-S-P. Each time a consumer reaches to service facility and Recognize that the service system is in W-B-S-P, every consumer initiates impatient (renege) timer" X, it is exponential state

ISSN 1462 2815

with rate ξ. Where ϕ represents the rate of maintaining consumers from impatience (reneging) with the probability $\phi + \phi_1$=1 If the alternate service provider is present while the working-breakdown earlier the period *X* pass away, the consumer is processed with the rate μ_b. If the primary service provider back to work after its repair process earlier the period *X* pass away, the primary server starts again and the consumer is serviced by the rate of μ_a. But, if the timer "*X*" pass away while the primary server is being repaired, the consumer leaves the facility and never back again.

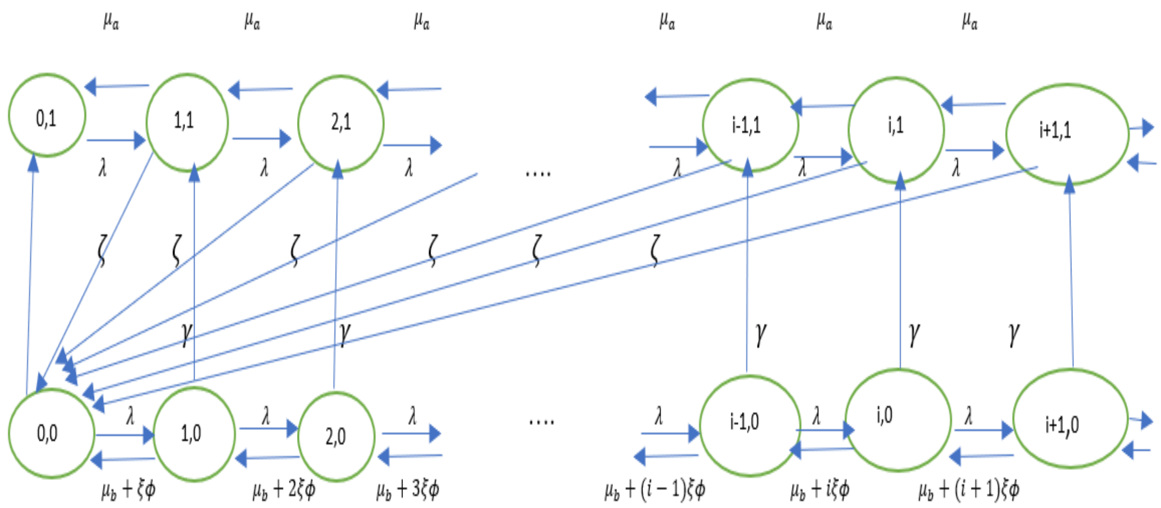

Figure 1: State-transition figure.

3. BALANCE EQUATIONS

In this part, the balance equations are established**,** Let *I*(*t*) be the number of consumers in the system and *K*(*t*) is the service provider's state

$$K(t) = \begin{cases} 0 & \text{primary server is in repair process at time t} \\ 1 & \text{primary server is under working at time t} \end{cases}$$

Then, the discrete-state process $\{(I(t);\ K(t));\ t \geq 0\}$ of a continuous-time turn into the Markov chain with state space $M = \{(i,\ m);\ n \geq 0;\ m = 0,1\}$. Figure 1 represents the state-transition illustration of the system. Let $P_{i,m} = \lim_{t\to\infty} P(I(t) = i;\ K(t) = m)$ symbolize the stationary probabilities of the stochastic Markov process $\{(I(t);\ K(t));\ t \geq 0\}$.

Then, the corresponding balance equations of the queueing system is,

$$(\gamma + \lambda(1+\varphi))p_{0,0} = (\mu_b + \xi\phi)p_{1,0} + \zeta \sum_{i=1}^{\infty} p_{i,1}, \qquad (1)$$

$$(\lambda(1+\varphi) + \gamma + \mu_b + i\xi\phi)p_{i,0} = [\mu_b + (i+1)\xi\phi]p_{i+1,0} + \lambda(1+\varphi)p_{i-1,0} \ ,i \geq 1 \qquad (2)$$

$$\lambda(1+\varphi)p_{0,1} = \mu_a p_{1,1} + \gamma p_{0,0} \qquad (3)$$

$$(\lambda(1+\varphi) + \zeta + \mu_a)p_{i,1} = \mu_a p_{i-1,1} + \lambda(1+\varphi)p_{i-1,1} + \gamma p_{i,0} \quad ,i \geq 1, \qquad (4)$$

define the Probability generating functions

$$P_0(z) = \sum_{n=0}^{\infty} z^i p_{i,0}, \quad P_1(z) = \sum_{i=1}^{\infty} z^i p_{i,1}$$

With $P_0(1) + P_1(1) + p_{0,1} = 1$ and $P_0'(z) = \sum_{i=1}^{\infty} iz^{i-1} p_{i,0}$ Product the equations (1) and (2) by 1,z^i,representing, Adding over "i" and reordering the parts, we get

$$\xi\phi z(1-z)P_0'(z) - [\lambda(1+\varphi) * z(1-z) - \mu_b(1-z) + \gamma z]P_0(z) = \mu_b(1-z)p_{0,0} - \zeta z P_1(1) \quad (5)$$

If z≠1 and z≠ 0 (5) are able to be expressed as.

$$P_0'(z) = \frac{[\lambda(1+\varphi)z(1-z) - \mu_b(1-z) + \gamma z]P_0(z) + \mu_b(1-z)p_{0,0} - \zeta z P_1(1)}{z\xi\phi(1-z)}. \quad (6)$$

Similarly, we got from (3)-(4)

$$[\lambda(1+\varphi) * z(1-z) - \mu_a(1-z) + \zeta z]P_1(z) = -\lambda(\varphi+1)z(1-z)p_{0,1} + zP_o(z)\gamma. \quad (7)$$

We get

$$P_1(z) = \frac{-\lambda(1+\varphi) * (1-z)zp_{0,1} + \gamma z P_o(z)}{\lambda(1+\varphi) * z(1-z) - \mu_a(1-z) + \zeta z}. \quad (8)$$

put z=1 in (8), we obtain

$$\gamma P_0(1) = \zeta P_1(1) \quad (9)$$

Remark 3.1. The existing queueing system becomes to M/M/1 queue with working-breakdown if we put $\varphi = 0, \phi = 0$ or ξ=0. This is a particular case of reference [10].

differential equation's solution,

Both the sides multiplying equation (6) by $e^{\int\left[\frac{-\lambda(1+\varphi)}{\xi\phi} + \frac{\mu_b}{\xi\phi x} - \frac{\gamma}{\mu_b(1-z)}\right]dz} = ce^{\frac{-\lambda(1+\varphi)}{\xi\phi}z}(1-z)^{\frac{\gamma}{\xi\phi}} z^{\frac{\mu_b}{\xi\phi}}$, where "c" is a constant term,

$$\frac{d}{dz}\left[e^{\frac{-\lambda(1+\varphi)}{\xi\phi}z}(1-z)^{\frac{\gamma}{\xi\phi}} z^{\frac{\mu_b}{\xi\phi}} P_0(z)\right] = \frac{(1-z) * \mu_b p_{0,0} - \zeta z P_1(1)}{\xi\phi z(1-z)} e^{\frac{-\lambda(1+\varphi)}{\xi\phi}z}(1-z)^{\frac{\gamma}{\xi\phi}} z^{\frac{\mu_b}{\xi\phi}} \quad (10)$$

Integrating both the sides of equation (10) from limit 0 to z and then rearranging the entire terms implies

$$P_0(Z) = (1-z)^{\frac{-\gamma}{\xi\phi}} z^{\frac{-\mu_b}{\xi\phi}} \int_0^z \frac{(1-t)\mu_b p_{0,0} - \zeta t P_1(1)}{(1-t)t} e^{\frac{\lambda(1+\varphi)}{\xi\phi}*(z-t)} (1-t)^{\frac{\gamma}{\xi\phi}} t^{\frac{\mu_b}{\xi\phi}} dt \quad (11)$$

$$R = \int_0^1 e^{\frac{\lambda(1+\varphi)}{\xi\phi}(1-t)} (1-t)^{\frac{\gamma}{\xi\phi}} * t^{\frac{\mu_b}{\xi\phi}-1} dt$$

$$J = \int_0^1 e^{\frac{\lambda(1+\varphi)}{\xi\phi}(1-t)} (1-t)^{\frac{\gamma}{\xi\phi}-1} * t^{\frac{\mu_b}{\xi\phi}} dt.$$

Take limit z→1 in equation (11), we get

$$P_0(1) = \left[\frac{\mu_b}{\xi\phi} p_{0,0} R - \frac{\zeta}{\xi\phi} P_1(1) J\right] \lim_{z\to 1} (1-z)^{\frac{-\gamma}{\xi\phi}},$$

Since $P_0(1) = \sum_{i=0}^{\infty} p_{i,0} < 1\ and\ \lim_{z\to 1}(1-z)^{\frac{-\gamma}{\xi\phi}} = \infty$, we should have that

$$\frac{\mu_b}{\xi\phi} p_{0,0} R - \frac{\zeta}{\xi\phi} P_1(1) J = 0. \qquad (12)$$

From equation (12), we get

$$P_1(1) = \frac{\mu_b R}{\zeta J} p_{0,0.} \qquad (13)$$

Substituting eq (13) into the eq (11) we get

$$P_0(z) = \frac{\mu_b}{\xi\phi} p_{0,0} (1-z)^{\frac{-\gamma}{\xi\phi}} z^{\frac{-\mu_b}{\xi\phi}} \int_0^z \frac{J-(R+J)t}{J} e^{\frac{\lambda(\varphi+1)}{\xi\phi}(z-t)} (1-t)^{\frac{\gamma}{\xi\phi}-1} t^{\frac{\mu_b}{\xi\phi}-1} dt. \quad (14)$$

Take the denominator portion of $P_1(z)$, we state $\eta(z) = \lambda(\varphi+1)z(1-z) + \zeta z$. Since

$$\eta(0) = -\mu_a < 0, \qquad \eta(1) = \zeta > 0, \qquad \eta(+\infty) < 0.$$

The roots of $\eta(z)$= 0 is present in (0, 1) and (1, +∞). Therefore $\eta(z)$= 0 it just has one root amid 0 and 1. Take z_1 be the root. Since $P_1(z)$ ≥ 0 for the interval 0 ≤ z ≤ 1, Numerator term of $P_1(z)$ should diminish at z=z_1. From equation (8) we have

$$p_{0,1} = \frac{\gamma P_0(z_1)}{\lambda(1+\varphi) * (1-z)} \qquad (15)$$

By the normalizing condition $1 = p_{0,1} + P_0(1) + P_1(1)$, we have

$$p_{0,0} = \frac{\lambda(1+\varphi) * \xi\phi\gamma\delta J}{\mu_b[\lambda(1+\varphi) * \xi\phi(\delta+\gamma)U + \zeta\gamma^2 J x(z_1)]}. \qquad (16)$$

where

$$x(z_1) = (1-z)^{\frac{-\gamma}{\xi\phi}-1} z_1^{\frac{-\mu_b}{\xi\phi}} \int_0^{z_1} \left(1 - \frac{R+J}{J} t\right) e^{\frac{\lambda(1+\varphi)}{\xi\phi}(z_1-t)} (1-t)^{\frac{\gamma}{\xi\phi}-1} t^{\frac{\mu_b}{\xi\phi}-1} dt.$$

The $p_{i,0}(i \geq 1)$ and $p_{i,1}(i \geq 1)$ are the probabilities, those can estimated in parts of $p_{0,0}$.

$$p_{i+1,0} = \frac{\lambda(1+\varphi) + \gamma + \mu_b + i\xi\phi}{\mu_b + (i+1)\xi\phi} p_{i,0} - \frac{\lambda(1+\varphi)}{\mu_b + (i+1)\xi\phi} p_{i-1,0}, \qquad i \geq 2,$$

$$p_{1,0} = \frac{(\lambda(1+\varphi)+\gamma)J - \mu_b R}{(\mu_b + \xi\phi)J} p_{0,0}.$$

Where, $p_{0,0}$ stated in equation (16),

$$p_{i+1,1} = \frac{\lambda(1+\varphi)+\zeta+\mu_a}{\mu_a} p_{i,1} - \frac{\lambda(1+\varphi)}{\mu_a} p_{i,1,0} - \frac{\lambda(1+\varphi)}{\mu_a} p_{i,0}, \qquad i \geq 1,$$

and $p_{0,0}$ is stated in equation (15).

Remark 3.2. The inequality $\zeta > 0$ is essential and sufficient state for the queue is stable in this study. This outcome can be gotten in Kim & Lee [10]. In actuality, every consumer is flushed out of the system whenever a breakdown arrives, It refers to the number of consumers at random epochs doesn't go to infinity.

4. ENCOURAGED SYSTEM-SIZE PROBABILITIES

We develop the probabilities of system-size in this part. The probability the primary service provider is at repair and working, represent by P_0, P_1, respectively, is given by

$$P_0 = P_0(1) = \frac{\mu_b R}{\gamma J} p_{0,0},$$

$$P_1 = P_1(1) + p_{0,1} = 1 - \frac{\mu_b R}{\gamma J} p_{0,0}.$$

Let $E(L_b)$, $E(L_a)$ indicate the expected number of system consumers after the primary server is repaired and functioning, respectively. put z→1 and apply L' Hospital rule in the right part of the eq (6), the term for $E(L_b)$ is attained as

$$E(L_b) = P_0'(1) = \frac{(\lambda(1+\varphi)-\mu_b)R+\gamma J}{\gamma(\xi\phi+\gamma)J} \mu_b p_{0,0}.$$

Differentiate "z" on both parts of eq (7) and take z=1 produce E (L_a) as

$$E(L_a) = \frac{\lambda(1+\varphi)-\mu_a}{\zeta} P_1(1) + \frac{\gamma}{\zeta} P_0'(1) + \frac{\lambda(1+\varphi)}{\zeta} P_{0,1}$$

$$= \frac{\lambda(1+\varphi)-\mu_a}{\zeta^2} \frac{R}{J} \mu_b p_{0,0} + \frac{\gamma x(z_1)}{\xi\phi\zeta} \mu_b p_{0,0} + \frac{\gamma}{\zeta} E(L_b).$$

Hence, the expected numbers of consumers in the service facility (system), meant by E(L_s),

$$\mathrm{E}(L_s) = E(L_b) + E(L_a)$$
$$= \left(\frac{\lambda(1+\varphi)-\mu_a}{\zeta^2} \frac{R}{J} + \frac{\gamma k(z_1)}{\xi\phi\delta} + \frac{\zeta+\gamma}{\zeta} \frac{(\lambda(1+\varphi)-\ \mu_b)R+\gamma J}{\gamma(\gamma+\xi\phi)J}\right) \mu_b * p_{0,0}$$

The expected reneging rate is

$$E(\xi) = \sum_{i=1}^{\infty} i\xi p_{i,0} = \xi E(L_b)$$

5. SOJOURN-TIME

Our consideration to the waiting-time or sojourn-time of a trial consumer (T-C) in this section, Where T-C is the total amount of time that has passed between arrival and departure from the system, whether due to an abandonment, breakdown, or the end of service. Let $W_{i,j}$ represent the sojourn-time of the trial consumer, upon consumer arrival he witnesses the state (i, j); i ≥ 0; j = 0, 1. Take d_1 denotes the interval of time between the arrival of the T-C and the next breakdown arrival. It is evident that its distribution is the same as the “d”. Thus the conditional expected waiting-time of the T-C incoming to the service facility in the (0, 1)th state follow as

$$E(W_{0,1}) = E(min\{V_1, d_1\}) = \frac{1}{\mu_a + \zeta}. \tag{17}$$

Undertake that the test consumer arrives at the (i,1) state of the system. Let X represents the period of the incomplete work instantly later the T-C arrival epoch. Then residual regular service-period plus the total of “i” usual service times is thus equal to X. Because of the memoryless-property, take notice that the residual regular service-period is stochastically equivalent to a new usual service. So, now we have

$$E(W_{i,1}) = E(min\{x, d_1\}) = \frac{(\mu_a + \zeta)^{i+1} - \mu_a^{i+1}}{\delta(\mu_1 + \zeta)^{i+1}} \tag{18}$$

Let's say the T-C is in state (i, 0) when it joins the system. Conditioning for the following event, we notice,

$$E(W_{i,0}) = \frac{1}{\lambda(1+\varphi) + \mu_b + \gamma + (i+1)\xi\phi} + \frac{\lambda(\varphi+1)}{\lambda(1+\varphi) + \mu_b + \gamma + (i+1)\xi\phi} E(W_{i,0})$$
$$+ \frac{\mu_b}{\lambda(1+\varphi) + \mu_b + \gamma + (1+i)\xi\phi} E(W_{i-1,0}) + \frac{\gamma}{\lambda(1+\varphi) + \mu_b + \gamma + (1+i)\xi\phi} E(W_{i,1})$$
$$+ \frac{(1+i)\xi\phi}{\lambda(1+\varphi) + \mu_b + \gamma + (1+i)\eta} \left(\frac{1}{i} \times 0 + \frac{i}{1+i} E(W_{i-1,0}) \right),$$

This term can be additional rewritten as

$$E(W_{i,0}) = \frac{1 + (\nu\mu_b + i\xi\phi)E(W_{i-1,0}) + \gamma E(W_{i,1})}{\nu\mu_b + \gamma + (i+1)\xi\phi} \tag{19}$$

We also have

$$E(W_{0,0}) = \frac{1}{\lambda(1+\varphi) + \gamma + \xi\phi} + \frac{\lambda(\varphi+1)}{\lambda(1+\varphi) + \gamma + \xi\phi} E(W_{0,0})$$
$$+ \frac{\gamma}{\lambda(1+\varphi) + \gamma + \xi\phi} E(W_{0,1}),$$

Implying that

$$E(W_{0,0}) = \frac{\mu_a + \zeta + \gamma}{(\mu_a + \zeta)(\gamma + \xi\phi)} \tag{20}$$

By iterating equation (19) and utilizing equation (18) and (20), we get, for $i \geq 1$

$$E(W_{i,0}) = I_{i+1} + \sum_{l=2}^{i} \prod_{i=l}^{i} \frac{\mu_b + i\eta}{\mu_b + \gamma + (i+1)\xi\phi} I_l + \prod_{i=1}^{i} \frac{\mu_b + i\xi\phi}{\mu_b + \gamma + (i+1)\xi\phi} E(W_{0,0})$$

$$+\frac{1}{\lambda(1+\varphi)+\mu_b+\gamma+(i+1)\eta}\left(1+\sum_{l=2}^{i}\prod_{i=l}^{i}\frac{\mu_b+i\xi\phi}{\mu_b+\gamma+i\xi\phi}\right),$$

Where

$$I_l=\frac{\gamma[(\zeta+\mu_a)^l-\mu_a^l]}{\zeta(\zeta+\mu_a)^{l+1}[\nu\mu_b+\gamma+(l+1)\xi\phi]}$$

And the conventions that $\sum_{l=2}^{i}a_l=0\ for\ i=1$, Lastly, we attain the expected sojourn-time of the T-C as

$$E(W_s)=\sum_{i=0}^{\infty}p_{i,0}E(W_{i,0})+\sum_{i=0}^{\infty}p_{i,1}E(W_{i,1}).$$

However, V_{served} is the most essential metric of system performance, determining the overall sojourn time of a consumer who has finished their service. Let $V_{i,j}$ the holding duration of a T-C who doesn't depart the service facility before finishing his task, assumed that upon consumer arrival is state (*i, j*). Then, *i* = 0, j=1.

$$E(V_{0,1})=P(V_1<d_1)E(V_1|s_1<V_1)=\frac{\mu_a}{(\mu_a+\zeta)^2}.$$

and for $i\geq 1$,

$$E(V_{i,1})=\frac{\mu_a}{\mu_a+\delta}\left(\frac{1}{\mu_a+\zeta}+E(V_{i-1,1})\right). \quad (21)$$

Iterating equation (21) we get

$$E(V_{i,1})=\frac{\mu_a[(\mu_a+\delta)^{i+1}-\mu_a^{i+1}]}{\zeta(\mu_a+\zeta)^{i+2}}. \quad (22)$$

Now, to analyze $E(V_{0,i})$ for i=0,1,2, …

$$E(V_{0,0})=\frac{\mu_b}{(\mu_b+\gamma+\xi\phi)^2}+\frac{\gamma}{\mu_b+\gamma+\xi\phi}\left(\frac{1}{\mu_b+\gamma+\xi\phi}+E(V_{0,1})\right), \quad (23)$$

For $i\geq 1$,

$$E(V_{i,0})=\frac{\nu\mu_b}{\mu_b+\gamma+(i+1)\xi\phi}\left(\frac{1}{\mu_b+\gamma+(i+1)\xi\phi}+E(V_{i-1,0})\right)$$
$$+\frac{\gamma}{\mu_b+\gamma+(i+1)\xi\phi}\left(\frac{1}{\mu_b+\gamma+(i+1)\xi\phi}+E(V_{i,0})\right)$$
$$+\frac{(1+i)\xi\phi}{\mu_b+\gamma+(i+1)\xi\phi}\frac{i}{i+1}\left(\frac{1}{\mu_b+\gamma+(i+1)\xi\phi}+E(V_{i-1,0})\right). \quad (24)$$

By iterating (24) and utilizing (22) produces

$$E(V_{i,0})=\alpha_i+\sum_{l=2}^{i}\alpha_{l-1}\prod_{i=l}^{i}\frac{\mu_b+i\xi\phi}{\mu_b+\gamma+(i+1)\xi\phi}+\prod_{i=1}^{i}\frac{\mu_b+i\xi\phi}{\mu_b+\gamma+(i+1)\xi\phi}E(V_{0,0})$$

Where

$$\alpha_l = \frac{\mu_b + l * \xi\phi}{(\mu_b + \gamma + (l+1)\xi\phi)^2} + \mu_a I_l$$

Lastly, the expected sojourn-time of the T-C that is assisted might be estimated utilizing the term

$$E(W_s) = \sum_{i=0}^{\infty} p_{i,0} E(V_{i,0}) + \sum_{i=0}^{\infty} p_{i,1} E(V_{i,1}).$$

6. NUMERICAL ILLUSTRATIONS

In this section, we provide numerical examples

Table 1: Represents the effect of λ vs P_{00} with φ=10% or 0.1, μ_a=2.0, ζ=1.5, γ=1.6, ξ=1, $\mu_b = 0.5, 1.0$ and 1.5, $\phi = 10\%$ or 0.1.

λ	μ_b	μ_b	μ_b
1.6	0.1722	0.1818	0.1869
1.7	0.1739	0.1843	0.19
1.8	0.1755	0.1867	0.1929
1.9	0.1771	0.1891	0.1957
2	0.1787	0.1913	0.1984
2.1	0.1804	0.1935	0.201
2.2	0.182	0.1956	0.2034
2.3	0.1836	0.1977	0.2057
2.4	0.1852	0.1997	0.208
2.5	0.1868	0.2016	0.2101
2.6	0.1884	0.2035	0.2122
2.7	0.1889	0.2053	0.2142
2.8	0.1915	0.2071	0.2161
2.9	0.193	0.2088	0.2179
3	0.1945	0.2105	0.2197

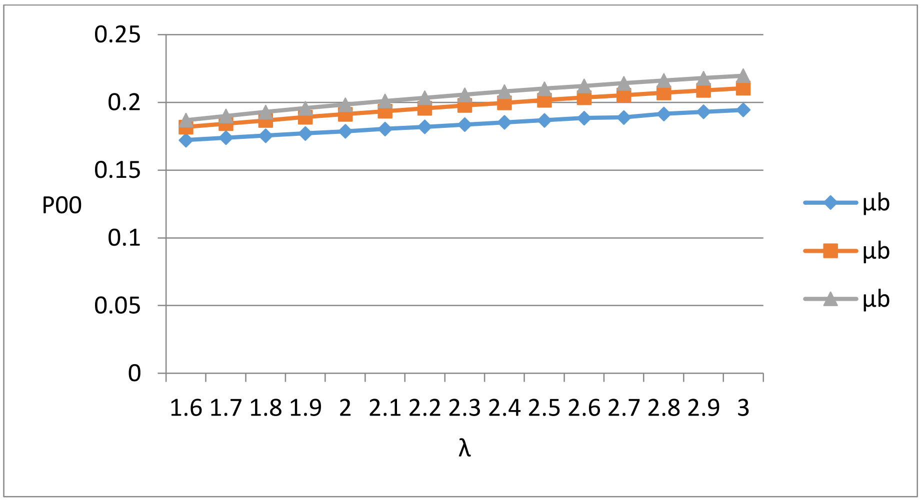

Figure 2: Represents the effect in λ vs P_{00} with φ=10%

Table 2: Represents the effect of λ vs E(L_s) with φ=10% or 0.2, μ_a=2.0, ζ=1.5, γ=1.6, ξ=1, $\mu_b = 0.5, 1.0$ and 1.5, $\phi = 0.1$.

λ	μ_b	μ_b	μ_b
1.6	0.7607	0.7038	0.6521
1.7	0.8319	0.7733	0.72
1.8	0.9032	0.8432	0.7886
1.9	0.9747	0.9135	0.8578
2	1.0463	0.9842	0.9274
2.1	1.1183	1.0552	0.9976
2.2	1.1899	1.1266	1.0682
2.3	1.262	1.1982	1.1392
2.4	1.3342	1.2701	1.2106
2.5	1.4066	1.3423	1.2823
2.6	1.4792	1.4147	1.3544
2.7	1.5519	1.4874	1.4268
2.8	1.6248	1.5603	1.4995
2.9	1.6979	1.6335	1.5725
3	1.7711	1.7069	1.6457

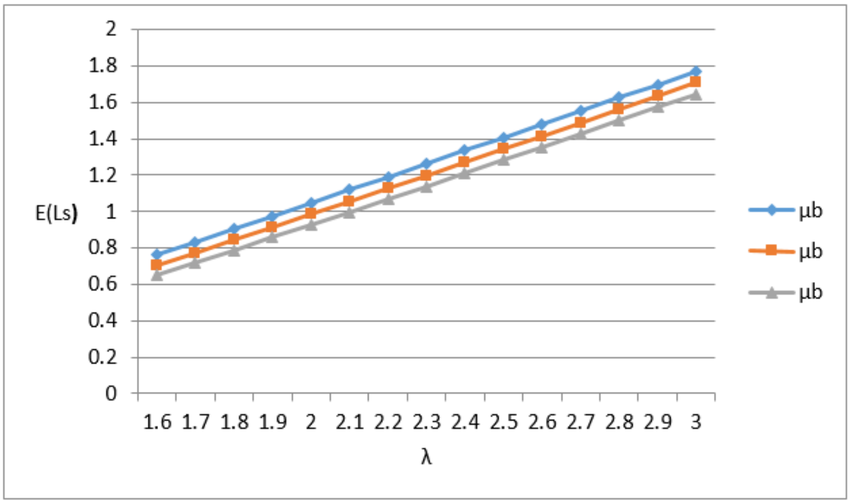

Figure 3: Represents the effect in λ vs E(Ls) with φ=10%

Table 3: Provides the variation in λ vs E(W_s) with φ=10% or 0.2, μ_a=1.8, ζ=1.5, γ=1.6, ξ=1, $\mu_b = 0.5, 1.0$ and 1.5, $\phi = 0.1$.

λ	μ_b=0.5	μ_b=1	μ_b=1.5
1.6	0.4322	0.3999	0.3705
1.7	0.4449	0.4135	0.385
1.8	0.4562	0.4259	0.3983
1.9	0.4663	0.4371	0.4104
2	0.4756	0.4474	0.4216
2.1	0.484	0.4568	0.4319
2.2	0.4917	0.4655	0.4414
2.3	0.4988	0.4736	0.4503
2.4	0.5054	0.4811	0.4585
2.5	0.5115	0.4881	0.4663
2.6	0.5172	0.4947	0.4736
2.7	0.5225	0.5008	0.4804
2.8	0.5275	0.5066	0.4868
2.9	0.5322	0.5121	0.4929
3	0.5367	0.5172	0.4987

ISSN 1462 2815

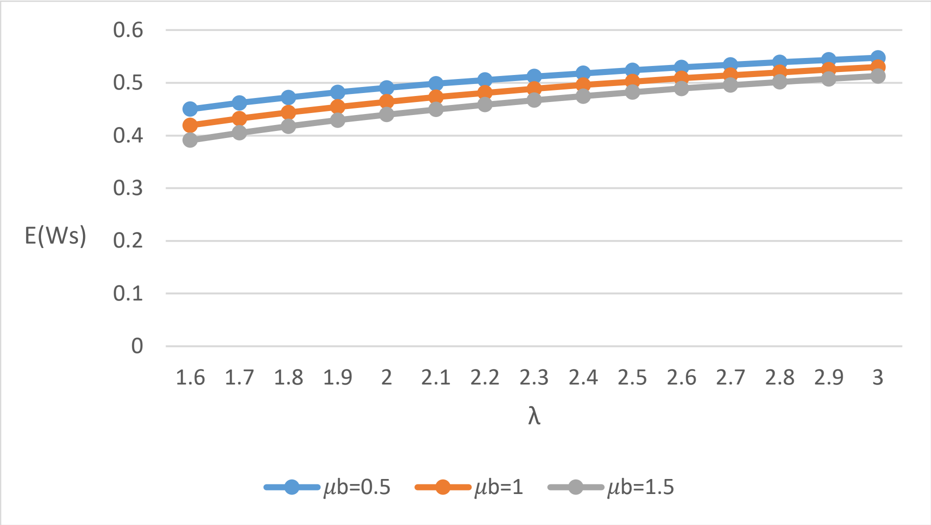

Figure 4: Represents the effect of sojourn time in λ vs E(W_s) with φ=10%

Table 4: Represents the effect of λ vs P_{00} with φ=20% or 0.2, μ_a=2.0, ζ=1.5, γ=1.6, ξ=1, $\mu_b = 0.5, 1.0$ and 1.5, $\phi = 0.1$.

λ	μ_b =0.5	μ_b=1	μ_b=1.5
1.6	0.1746	0.1854	0.1913
1.7	0.1764	0.188	0.1945
1.8	0.1781	0.1905	0.1975
1.9	0.1799	0.1929	0.2003
2.0	0.1817	0.1952	0.203
2.1	0.1835	0.1975	0.2055
2.2	0.1852	0.1997	0.208
2.3	0.1869	0.2018	0.2103
2.4	0.1886	0.2038	0.2125
2.5	0.1903	0.2058	0.2147
2.6	0.1920	0.2077	0.2168
2.7	0.1937	0.2095	0.2188
2.8	0.1953	0.2114	0.2207
2.9	0.1969	0.2131	0.2225
3.0	0.1985	0.2148	0.2243

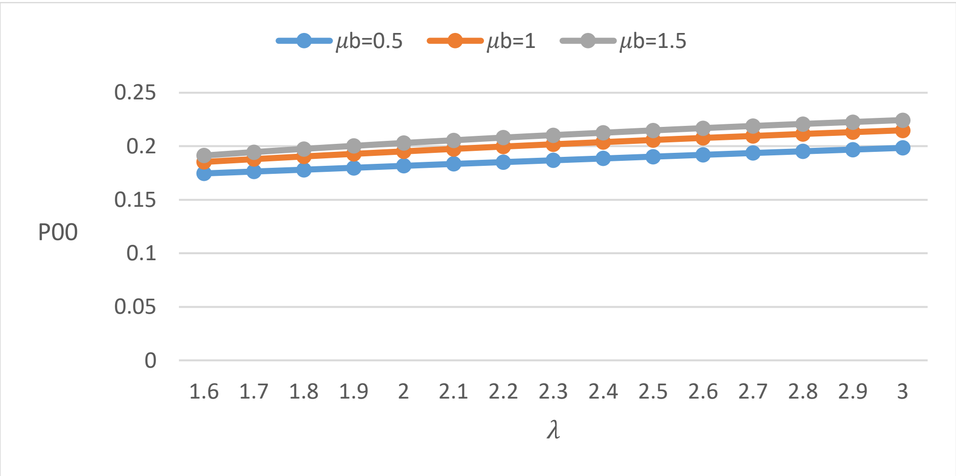

Figure 5: Represents the effect in λ vs P_{00}

Table 5: Provides the effect in λ vs E(Ls) with φ=20% or 0.2, μ_a=2.0, ζ=1.5, γ=1.6, ξ=1, $\mu_b = 0.5, 1.0$ and 1.5, $\phi = 0.1$.

λ	μ_b =0.5	μ_b=1	μ_b=1.5
1.6	0.8643	0.805	0.7511
1.7	0.9422	0.8815	0.8263
1.8	1.0202	0.9585	0.902
1.9	1.0984	1.0358	0.9784
2	1.1768	1.1136	1.0553
2.1	1.2554	1.1917	1.1327
2.2	1.3342	1.2701	1.2106
2.3	1.4132	1.3489	1.2889
2.4	1.4924	1.4279	1.3675
2.5	1.5718	1.5073	1.4466
2.6	1.6514	1.5869	1.526
2.7	1.7311	1.6668	1.6057
2.8	1.8111	1.747	1.6858
2.9	1.8912	1.8274	1.7661
3	1.9715	1.908	1.8467

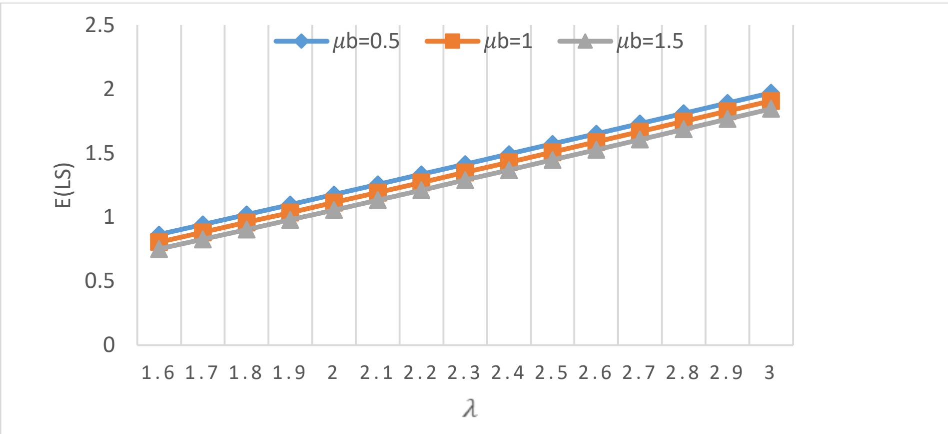

Figure 6: The effect of E(L_s) when λ vs E(Ls) with φ=20%

Table 6: Provides the variation in λ vs E(W_s) with φ=20% or 0.2, μ_b=1.8, ζ=1.5, γ=1.6, ξ=1, $\mu_a = 0.5, 1.0$ and 1.5, $\phi = 0.1$.

λ	μb=0.5	μb=1	μb=1.5
1.6	0.4501	0.4193	0.3912
1.7	0.4618	0.4321	0.405
1.8	0.4723	0.4437	0.4176
1.9	0.4818	0.4543	0.4291
2	0.4904	0.464	0.4397
2.1	0.4982	0.4729	0.4495
2.2	0.5054	0.4811	0.4585
2.3	0.512	0.4887	0.467
2.4	0.5182	0.4958	0.4748
2.5	0.5239	0.5024	0.4822
2.6	0.5293	0.5086	0.4891
2.7	0.5343	0.5144	0.4956
2.8	0.539	0.5199	0.5017
2.9	0.5435	0.5251	0.5075
3	0.5476	0.53	0.513

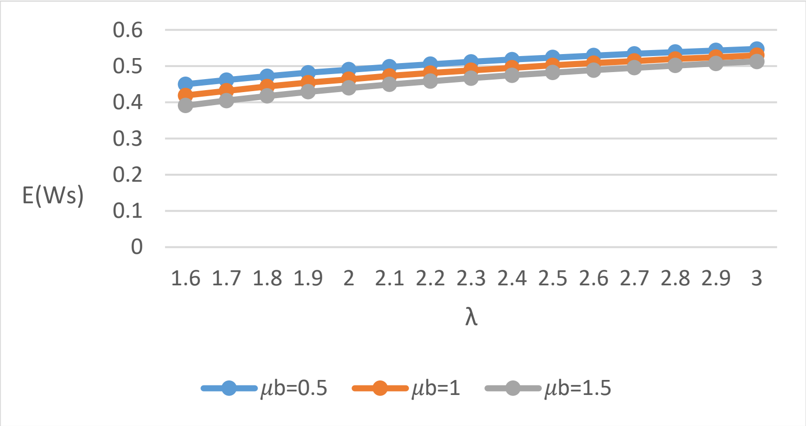

Figure 7: Represents the effect of sojourn time in λ vs E(W$_s$)

Table 7: Provides the variation in ζ vs E(L$_s$) with φ=20% or 0.2, μ_b=2.0, $\lambda = 2$, ζ=0.4 to 2.2, γ=1.6, ξ=1.6, $\mu_a = 1.5$, $\phi = 0.1$.

ζ	ξ=1.5	ξ=1	ξ=0.5
0.4	1.2238	2.2979	2.8859
0.5	1.0159	2.0177	2.5692
0.6	0.8898	1.7626	2.3211
0.7	0.8077	1.579	2.1221
0.8	0.7516	1.4402	1.9592
0.9	0.7116	1.3314	1.8234
01	0.6822	1.2438	1.7085
1.1	0.6602	1.1718	1.61
1.2	0.6433	1.1115	1.5246
1.3	0.6301	1.0603	1.45
1.4	0.6196	1.0162	1.3842
1.5	0.6113	0.9779	1.3257
1.6	0.6045	0.9444	1.2735
1.7	0.599	0.9148	1.2265
1.8	0.5945	0.8884	1.184

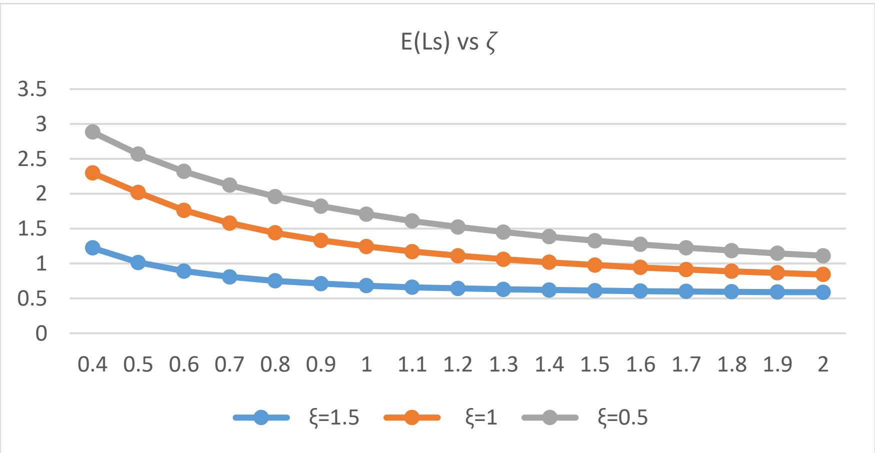

Figure 8: represents ζ vs E(L$_s$)

Table 8: Provides the variation in γ vs E(L_s) with $\lambda = 2$, φ=20% or 0.2, μ_a=1.8, 2, 2.5, ζ=1.6, γ=0.5 to 2, ξ=1.6, $\mu_b = 1.5$, $\phi = 0.1$.

γ	μ_a=1.8	μ_a=2.0	μ_a=2.5
0.5	0.3586	0.3573	0.3542
0.6	0.3394	0.3379	0.3341
0.7	0.3198	0.318	0.3137
0.8	0.3005	0.2986	0.2938
0.9	0.282	0.2799	0.2748
01	0.2644	0.2622	0.2569
1.1	0.2479	0.2457	0.2401
1.2	0.2326	0.2302	0.2246
1.3	0.2183	0.216	0.2103
1.4	0.2051	0.2027	0.197
1.5	0.1929	0.1905	0.1849
1.6	0.1816	0.1792	0.1736
1.7	0.1711	0.1688	0.1633
1.8	0.1615	0.1592	0.1537
1.9	0.1525	0.1503	0.1449

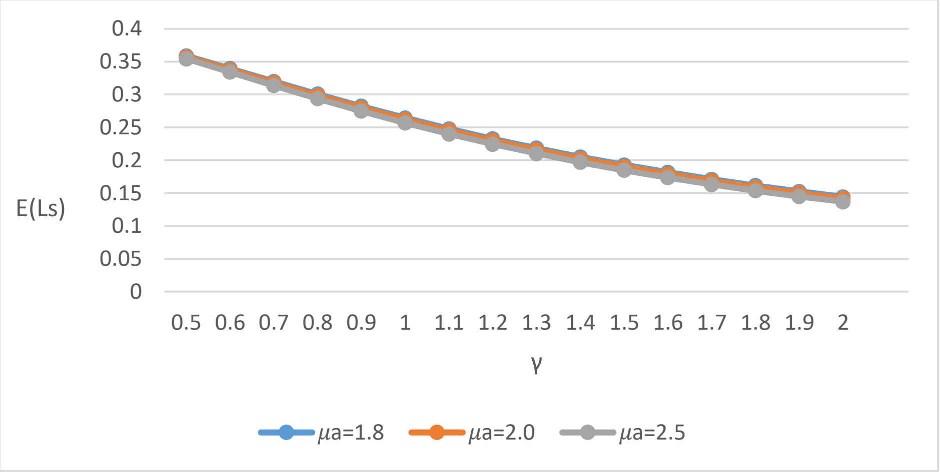

Figure 9: Represents the effect in γ vs E(L_s)

Table 9: Represents the effect in $\mu b/\mu a$ vs E(L_s) with $\lambda = 2$, φ=20% or 0.2, ζ=1.6, γ=0.5,1,1.5, ξ=1.6.

μ_b/μ_a	γ=0.5	γ=1	γ=1.5
1	0.5658	0.3686	0.2535
1.1	0.5396	0.3552	0.2456
1.2	0.5135	0.3419	0.2378
1.3	0.4875	0.3288	0.2301
1.4	0.4615	0.3157	0.2225
1.5	0.4357	0.3027	0.2149
1.6	0.4099	0.2899	0.2075
1.7	0.3842	0.2771	0.2001
1.8	0.3586	0.2644	0.1929
1.9	0.333	0.2518	0.1857
2	0.3076	0.2393	0.1785

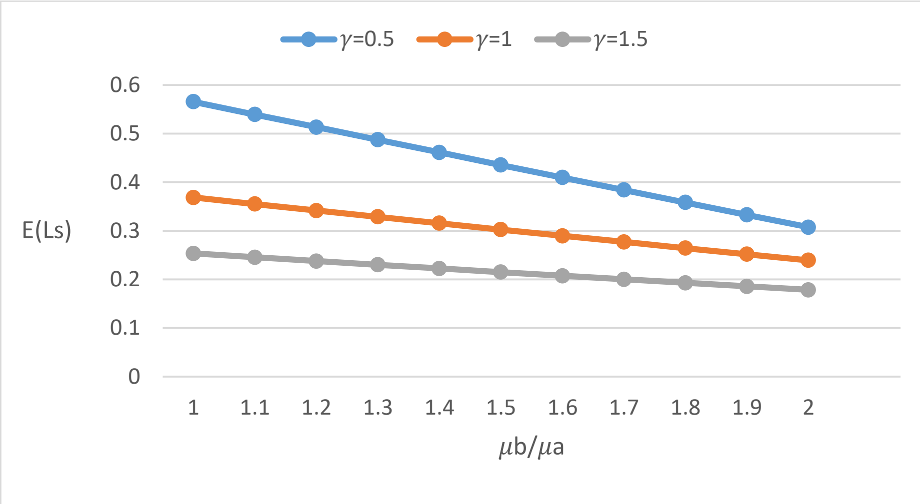

Figure 10: Represents μ_b/μ_a vs $E(L_s)$

Table 10: Represents the effect in ϕ vs $E(W_s)$ with $\lambda = 2$, φ=20% or 0.2, ζ=1.5, γ=1.6, ξ=0.5, $\mu_0 = 1.5$, ϕ=0 to 0.9, $\mu_b = 2.5$.

ϕ	$E(W_s)$
0	0.2705
0.1	0.2547
0.2	0.2367
0.3	0.2164
0.4	0.1937
0.5	0.1684
0.6	0.1401
0.7	0.1096
0.8	0.0759
0.9	0.0393

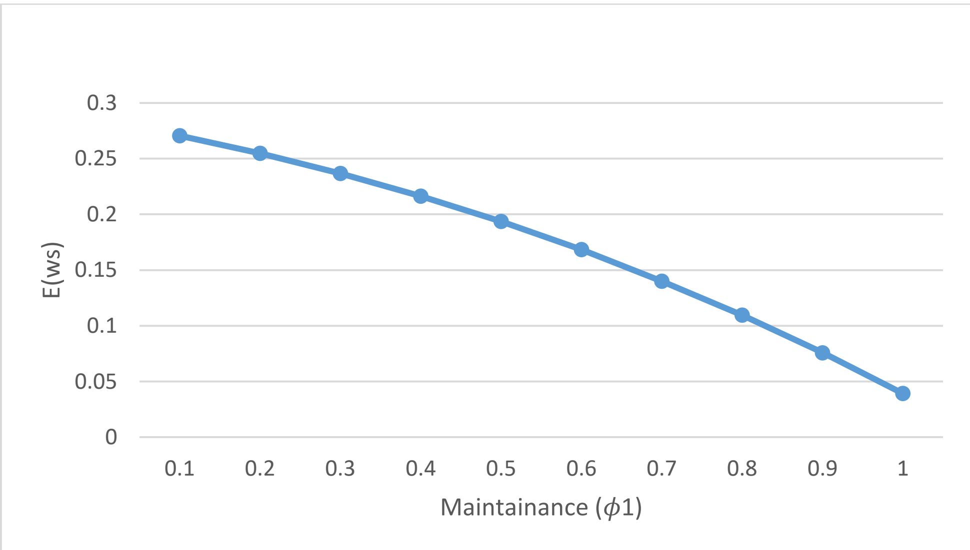

Figure 11: Represents the effect of ϕ_1 vs $E(W_s)$

7. RESULT AND DISCUSSIONS

In this part, we discuss the outcomes of numerical examples. Table 1 and figure 2 represents the increasing in P_{00} values when the arrival rate λ=1.6 to 3.0 with the discount rate $\varphi = 10\% \text{ or } 0.1$ and the different service rates of alternative server ($\mu_b = 0.5, 1.0, 1.5$).

When Table 2 and figure 3 demonstrates that the outcomes of $E(L_s)$ are highly increasing when we increase φ this made the $E(L_s)$ is higher than the Poisson arrival model figure 3 in [20]. Table 3 and figure 4 demonstrates that the outcomes of $E(W_s)$ are slightly increasing when we increase the arrival rate λ=1.6 to 3.0 with the maximum discount rate $\varphi = 10\% \text{ or } 0.1$ and the different service rates of alternate server ($\mu_b = 0.5, 1.0, 1.5$).

Table 4 and figure 5 represents the increasing in P_{00} values when the arrival rate λ=1.6 to 3.0 with the discount rate $\varphi = 20\% \text{ or } 0.2$ and the different service rates of alternate server ($\mu_b = 0.5, 1.0, 1.5$). when Table 5 and figure 6 demonstrates that the outcomes of $E(L_s)$ are highly increasing when we increase φ this made the $E(L_s)$ is higher than the Poisson arrival model figure 3 in [20].

Table 6 and figure 7 demonstrates that the outcomes of $E(W_s)$ are slightly increasing when we increase the arrival rate λ=1.6 to 3.0 with the maximum discount rate $\varphi = 20\% \text{ or } 0.2$ and the different service rates of alternate server ($\mu_b = 0.5, 1.0, 1.5$). Based on the outcome we obtained when $\varphi = 20\%$ (table 5) attained a lot of users to the system "($E(L_s)$)" than $\varphi = 10\%$ (table 2). The same concept with a different queuing mechanism were studied by [6, 7].

Table 7 and figure 8 represents that the outcomes of $E(L_s)$ are reducing when we increase the breakdown rate δ=0.4 to 2.0 with the different reneging rates of consumers ($\xi = 0.5, 1, 1.5$) due to the server crash (breakdown) i.e., (Here the primary server is under repair because of the breakdown). The implementation of encouraged consumer arrival and maintaining the consumers from impatience made the $E(L_s)$ is much higher than in (figure 6 (b) of ref [20]).

Table 8 and figure 9 shows that the outcomes of $E(L_s)$ are reducing when we increase the repair rate γ =1.6 to 3.0 with the different service rates of primary server ($\mu_a = 1.8, 2.0, 2.5$) due to the working-breakdown service policy i.e., (Here the primary server is under repair because of the breakdown). Usually, consumers mostly quit the service facility when the server gets breakdown. At $\xi = 1.5$ has the lowest system size than 0.5 and 1.0.

From Table 9, Figure 10, we can see the ratio of μ_a and μ_b, E(L) reduce when the rate of γ (repair) rise. Obviously, the growing of E(L) is depended on the μ_a and μ_b. Now, In table 10 and figure 11 reveals that when the rate of maintenance increases "$\phi_1 = 0.1 \text{ to } 0.9$" then the sojourn-time $E(W_s)$ decreasing simultaneously. Minimized the sojourn-time by maintaining consumers from impatient behavior.

8. CONCLUSION

In this article, we have developed the effectiveness of an M/M/1 queue with working breakdown policy, and maintenance of impatient consumers along with the proposed encouraged system-size analysis. We developed the balance equations by using the stochastic continuous time Markov-chain.

Performance metrics, such as the probabilities of the service provider's state, expected number of consumers in the system and expected sojourn-time of consumers are developed. By utilizing the encouraged arrival strategy, we maximized the system-size.

Reduced the sojourn-time by maintenance of impatient consumers. In a lot of real-life situations, when a business offers a discount to its consumers, the consumers are more encouraged to use the services of the company even if it is already overcrowded. The more we maintain the system during working-breakdown will reduce the chances of consumers to get impatient. We will extend this research by bulk-service queue or deterministic service queueing system in the future.

References

1) Altman, E., Yechiali.U. 2006. Analysis of customer's impatience in queues with server vacations. *Queueing Syst*., 52, 261-279.

2) Atencia, I., Moreno, P. 2004. The discrete-time Geo/Geo/1 queue with negative customers and disasters. *Comput. Oper. Res.,* 31, 1537-1545.

3) Choudhury, G., Tadj, L., Paul, M. 2007. Steady state analysis of an MX/G/1 queue with two phase service and Bernoulli vacation schedule under multiple vacation policy. *Applied Mathematical Modelling*, 31, 1079–1091.

4) Jiang, T., Liu, L., Li, J. 2015. Analysis of the M/G/1 queue in multi-phase random environment with disasters. *J. Math. Anal. Appl.*, 430 ,857-873.

5) Jeyachandhiran, R., Rajendran, P. 2024. Analysis of a Single Server Retrial Queueing System with Finite Capacity, F-Policy, Balking, Reneging and Server Maintenance. *Contemporary Mathematics*, 5, 1-1121.

6) Jeyachandhiran, R., Rajendran, P. 2023. Efficacy of the Single Server Markovian Two-Phase Symmetric Queue with Encouraged Stationary-Queue-Size Analysis. Contemp. Math. *Contemp. Math*., 4, 620-1346.

7) Khan, I.E., Paramasivam, R. 2022. Reduction in waiting time in an M/M/1/N encouraged arrival queue with feedback, balking and maintaining of reneged customers. Symmetry 14, 1743.

8) Khan, I.E., Paramasivam, R. 2023. Analysis of encouraged arrival multiple working vacation queueing model under the steadystate condition. *Reliability: Theory & Applications*, 18, 402-407.

9) Kalidass, K., Kasturi. 2012. R. A queue with working breakdowns. *Comput. Ind. Eng*. 63, 719-783.

10) Kim, B.K., Lee, D.H. 2014. The M/G/1 queue with disasters and working breakdowns. *Appl. Math. Model.* 38, 1788- 1798.

11) Laxmi, P.V., Jyothsna, K. 2015. Impatient customers queue with Bernoulli schedule vacation interruption. *Comput. Oper. Res.* 56, 1-7.

12) Lee, D.H., Yang, W.S., Park, H.M. Geo/G/1 queues with disasters and general repair times. *Appl. Math. Model.* 35, 1561-1570.

13) Perel, N., Yechiali, U. 2005. Queues with slow servers and impatient customer*. Eur. J. Oper. Res.* 201, 247-258.

14) Selvaraju, N., Goswami,C. 2013. Impatient customers in an M/M/1 queue with single and multiple working vacations. *Comput.Ind. Eng.* 65, 207-215.

15) Som, B.K., Seth, S. 2017. An M/M/1/N queuing system with encouraged arrivals. *Global Journal of Pure and Applied Mathematics*. 13, 3443-3453.

16) Sudhesh, R. 2010. Transient analysis of a queue with system disasters and customer impatience. *Queueing Syst*. 66, 95-105.

17) Towsley, D., Tripathi, S.K. 1991. A single server priority queue with server failures and queue flushing. *Oper. Res. Lett.* 10, 353-362.

18) Yechiali, U. 2007. Queues with systems disasters and impatient customers when system is down. *Queueing Syst.* 56, 195-202.

19) Yue, D., Yue, W., Xu, G. 2012. Analysis of customers impatience in an M/M/1 queue with working vcations. *J. Ind. Manage. Optim.*, 8, 895-908.

20) Zhang, M., Gao, H. 2020. The disasters queue with working breakdowns and impatient customers, *RAIRO-Oper. Res.* 54, 815-825.

21) Klebanov, L.B. 2023. Some More Results on Characterization of the Exponential and Related Distributions. *Stats*. *6*, 740-746.